Units, Nomenclature, and Environmental Corrections for P_e and Density Measurements

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Photo-Electric Factor

Units

The true definition of this measurement is proportional to the ratio of the photoelectric cross section to the Compton cross section[1], and is a unitless quantity. The lack of units is obvious in the approximation for the measurement that is generally used by the industry to compute factor for an element: $(Z/10)^{3.6}$. However, it is common in the industry to arbitrarily assign units of barns/electron to the measurement. The SIG should recommend that the measurement be treated as unitless.

Nomenclature

The measurement was introduced by Bertozzi et al. as a photoelectric index[2], and they represented it with the symbol P_e. Since then the measurement has sometimes been referred to as the photoelectric factor, and various authors have represented it by P_e, *Pe*, P_{ef} and *Pef*. It would benefit the industry if the SIG were to standardize the nomenclature. Below is a suggestion on how to do so.

Because the measurement is defined as a ratio of cross sections, it is not purely a measure of photoelectric absorption. Therefore, it is not proper to call it a photoelectric index or factor. It seems best to just refer to it by the letters “p e”.

When writing equations, it is usually clearer to use one symbol per variable. For example, the letters *Pe* could be construed as the variable P multiplied by the constant e (e = 2.71…). Thus, it makes sense to use the symbol P_e for the measurement when writing equations, where the “e” is a subscript. Although this confusion would not arise in normal text, using this nomenclature in sentences would avoid confusion with the letter “p”, which is spelled “pe”, although this is not likely to be a point of confusion.

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[1] Moake, G.L., 2011, Using Computer Modeling to Generate Accurate Pe Equations, SPWLA 52nd Annual Logging Symposium, paper MMM.

[2] Bertozzi, W., Ellis, D.V., and Wahl, J.S., 1981, The physical foundation of formation lithology logging with gamma rays, Geophysics, vol. 46, No. 10, p. 1439–1455.

However, it is always valuable to be consistent between symbols used in text and those used in equations. Thus, the SIG should recommend that P_e be used in both equations and text. Although formatting the subscript was once a pain, autocorrect features of modern word processors allow this to be entered easily. For example, in Microsoft Word 2010 it is possible to define a correction that replaces "P{" with "P_e ", where the blank in the substituted text is formatted to be normal and not an italicized subscript.

When referring to the P_e of specific materials, a subscript should be used to denote the material, and it should be separated from the "e" by a comma. For example, if one was referring to the P_e of marble, the symbol would be $P_{e,marble}$. There are five materials in the logging problem that are commonly referenced, and it would be useful to standardize their abbreviations. It is recommended that the following abbreviations be used.

P_e Nomenclature and Symbols

Material	*Subscript*	*P_e Symbol*
Formation matrix	ma	$P_{e,ma}$
Formation fluid	fl	$P_{e,fl}$
Generic formation	N/A	P_e
Mud	m	$P_{e,m}$
Mud filtrate	mf	$P_{e,mf}$
Mudcake	mc	$P_{e,mc}$

To further clarify that P_e is a variable, and to be consistent with many publications, it is also recommended that P_e be italicized.

Environmental Corrections

Although the P_e of materials does not change with temperature, the measurement can be affected by changes in the resolution of the detector. The P_e measurement is also effected by mud in the borehole, particularly barite mud. There are several causes for this effect. A mismatch in diameters between the tool and borehole wall results in a thin layer of mud through which many of the detected gamma rays must pass. In addition, mudcake or standoff may provide a much larger layer of barite-laden material that must be penetrated.

Traditionally, temperature changes have either been ignored or automatically applied by the service provider. Some attempts have been made at correcting P_e for standoff over the years, but the measurement is so weak in the presence of heavy muds that the corrections generally don't help much. Perhaps the best recommendation on environmental corrections is to automatically apply temperature and diameter/mud-weight corrections if the vendor has them. The application of standoff corrections should be optional, but the log should indicate that they have been made. If a standoff correction is made, a P_e correction curve should be available to indicate the size of the correction.

Density

The units of density are not in question, and there is no reason to change the accepted symbol (ρ) for density. However, there are some nomenclature and descriptive issues that can be addressed. It would be useful to assign standard abbreviations for common materials, and it makes sense to use the same abbreviations used for P_e. There is also the issue of identifying the type of density being used. As with P_e, it is recommended that ρ be italicized.

There are three different densities that are relevant to density logging, and there are times when it is important to distinguish between them. The *bulk density* is the mass per unit volume. Density tools respond to the *electron density*, which for a material composed of one type of molecule is defined as twice the number of protons in the molecule times the bulk density of the material divided by the atomic mass of the molecule. In an effort to provide a density value comparable to the bulk density, the *apparent bulk density* is defined as 1.0704 times the electron density minus 0.1883, i.e.

$$\rho_{apparent\ bulk\ density} = 1.0704\rho_{electron\ density} - 0.1883 \ .$$

Since the industry has been using ρ_b to represent the apparent bulk density, it seems reasonable to continue with that representation, despite its confusion with the true bulk density. The letter *k* can then be used for the bulk density, and the letter *e* for the electron density. The question becomes where to put them. The options discussed in this paragraph are illustrated in the equations below, which were generated using MathType 6.7a. If the descriptors are written as a superscript, then the designator could possibly be confused with a multiplicative power for the density; and when a multiplicative power is required, it becomes more awkward. If the descriptors are written to the left of ρ, it can be confusing because that is a nonstandard mathematical approach and is awkward to use in equation editors such as MathType. It seems that the best way is to write them as subscripts, and separate other descriptors from them by a comma, just as is done with P_e. The descriptor for the type of density should be written first.

$$\rho_m^e = \frac{2Z}{A}\rho_m^b \qquad x = Something \cdot \left(\rho_m^b\right)^2$$

$${}^e\rho_m = \frac{2Z}{A}{}^b\rho_m \qquad x = Something \cdot {}^b\rho_m^2$$

$${}_b\rho_m = \frac{2Z}{A}{}_b\rho_m \qquad x = Something \cdot {}_b\rho_m^2$$

$$\rho_{e,m} = \frac{2Z}{A}\rho_{b,m} \qquad x = Something \cdot \rho_{b,m}^2$$

The recommended nomenclature and symbol presentation for density are given in the table below.

Density Nomenclature and Symbols

Value / Material	*Subscript*	*Symbol*
True Bulk density	tb	ρ_{tb}
Electron density	e	ρ_e
Apparent bulk density	b	ρ_b
Formation matrix (generic density)	ma	ρ_{ma}
Formation fluid (generic density)	fl	ρ_{fl}
Generic formation (generic density)	N/A	ρ
Mud (generic density)	m	ρ_m
Mudcake (generic density)	mc	ρ_{mc}
Mud filtrate	mf	ρ_{mf}
Bulk density of the matrix	b,ma	$\rho_{b,ma}$
Electron density of the formation fluid	e,fl	$\rho_{e,fl}$
Apparent bulk density of the mud	b,m	$\rho_{b,m}$
Standoff/mudcake correction to the apparent bulk density	N/A	$\Delta\rho$

Using this nomenclature, the relationship between apparent bulk density and electron density is written as

$$\rho_b = 1.0704\rho_e - 0.1883 \ .$$

Environmental Corrections

The primary obstacle to computing an accurate formation density is standoff (or mudcake). Density tools are designed to compensate for this effect (although the correction is not perfect in the real world). Thus, the compensation is not really an environmental correction, since it is not an extra correction that must be applied to the log. Historically, the compensated density was computed by first computing the density from only the far detector and then adding a density correction, called DRho or $\Delta\rho$, to the far density to obtain the compensated density. More sophisticated approaches compute the density directly from multiple inputs; the traditional $\Delta\rho$ is then computed for use as a quality indicator. Regardless of how the processing is done, the final density is always compensated for standoff/mudcake.

The only significant environmental corrections that are relevant to the density tool are temperature, borehole diameter, and mud weight. Gain stabilization minimizes the need for a temperature correction, but resolution changes in the detector can still result in a small temperature drift. Curvature mismatch between the front of the density tool and the borehole wall can affect the measurement, with the magnitude of the effect depending on the mud properties. In addition, LWD tools may detect gamma rays that pass into the tool through the blades of the stabilizer instead of the front of the stabilizer, which also causes a mud dependence. In the past, caliper/mud corrections have been applied automatically by the vendor. Temperature corrections, if they are known, have also been applied automatically. It is recommended that all these corrections be applied automatically by the service company.